EC 3210 Solutions

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Assignment 1

1.1. Consider the lasers listed in the example on page 5. If the spectral linewidth of these lasers is 1 nm, ...

a. ... calculate the frequency linewidth $\Delta\nu$ for each source.

b. ... calculate the fractional linewidth $\Delta\nu/\nu$ for each laser.

c. ... calculate the Q of each source where Q is defined by $Q = \nu/\Delta\nu$.

Solution: We have a HeNe, CO_2, Nd:glass, and ruby laser. For a spectral linewidth $\Delta\lambda = 1$ nm, we find the following results using:

Laser	Wavelength	$\Delta\nu = \frac{c\,\Delta\lambda}{\lambda^2}$	$\Delta\nu/\nu = \frac{\Delta\lambda}{\lambda}$	Q $= \frac{\nu}{\Delta\nu}$
HeNe	632.8×10^{-9}	7.45×10^{11}	1.580×10^{-3}	632.8
CO_2	10.6×10^{-6}	2.67×10^{9}	9.43×10^{-5}	10600
Nd:glass	1.06×10^{-6}	2.67×10^{11}	9.43×10^{-4}	1060
Ruby	694×10^{-9}	6.23×10^{11}	1.441×10^{-3}	694

Note: This problem is easily done using a spreadsheet program.

1.2. Find the diameter of an extended source that can be considered a point source at wavelengths of 500 nm and 10 μm for

a. ... a lab bench that is one meter long.

b. ... the optical horizontal line of sight in the atmosphere (approximately 100 km).

c. ... for the earth–moon distance of 450,000 km.

Solution: See Figure 1 for the geometry of this problem. We want

$$R >> \frac{h^2}{\lambda}. \tag{1}$$

a. Choosing a factor of 10x, we have

$$R = 10\frac{h^2}{\lambda}, \tag{2}$$

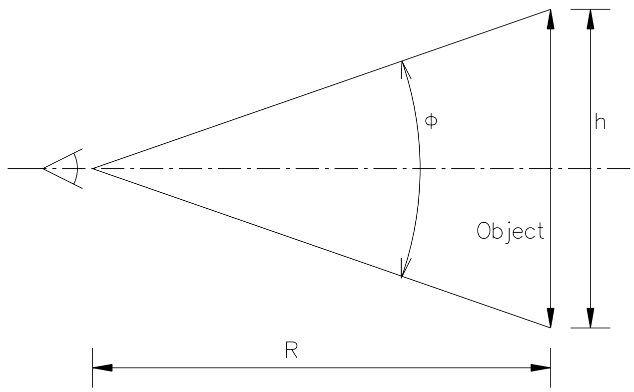

Figure 1: Geometry for Problem 1.2. Object is h high and located R away from observer.

so for $R = 1$ and $\lambda = 500$ nm,

$$h = \sqrt{\frac{R\lambda}{10}} = \sqrt{\frac{1(500 \times 10^{-9})}{10}} = 2.24 \times 10^{-4} \text{ m}\,. \tag{3a}$$

For $R = 1$ m and $\lambda = 10 \times 10^{-6}$ m,

$$h = \sqrt{\frac{R\lambda}{10}} = \sqrt{\frac{1(10 \times 10^{-6})}{10}} = 1 \times 10^{-3} \text{ m}\,. \tag{3b}$$

b. For $R = 100$ km $= 1 \times 10^5$ m and $\lambda = 500$ nm,

$$h = \sqrt{\frac{R\lambda}{10}} = \sqrt{\frac{(1 \times 10^5)(500 \times 10^{-9})}{10}} = 7.07 \times 10^{-2} \text{ m}\,. \tag{4a}$$

For $R = 1 \times 10^5$ m and $\lambda = 10 \times 10^{-6}$ m,

$$h = \sqrt{\frac{R\lambda}{10}} = \sqrt{\frac{(1 \times 10^5)(10 \times 10^{-6})}{10}} = 3.16 \times 10^{-1} \text{ m}\,. \tag{4b}$$

c. For $R = 450000$ km $= 4.5 \times 10^8$ m and $\lambda = 500$ nm,

$$h = \sqrt{\frac{R\lambda}{10}} = \sqrt{\frac{(4.5 \times 10^8)(500 \times 10^{-9})}{10}} = 4.73 \text{ m}\,. \tag{5a}$$

For $R = 4.5 \times 10^8$ m and $\lambda = 10 \times 10^{-6}$ m,

$$h = \sqrt{\frac{R\lambda}{10}} = \sqrt{\frac{(4.5 \times 10^8)(10 \times 10^{-6})}{10}} = 21.2 \text{ m}\,. \tag{5b}$$

1.3. The earth-moon distance is approximately 450,000 km. Calculate the diameter of a beam sent from the earth ...

a. ... if the full–angle laser beam divergence is 1 mr.

b. ... if the laser beam divergence is 1 μr?

c. Design a beam collimator to reduce the beam divergence from the value in part (a) to the value in part (b). Assume that the laser beam diameter in part (a) is 1 mm.

<u>Solution:</u> a. The spot size for a beam divergence of 1 mr (see Fig. 2) is given by

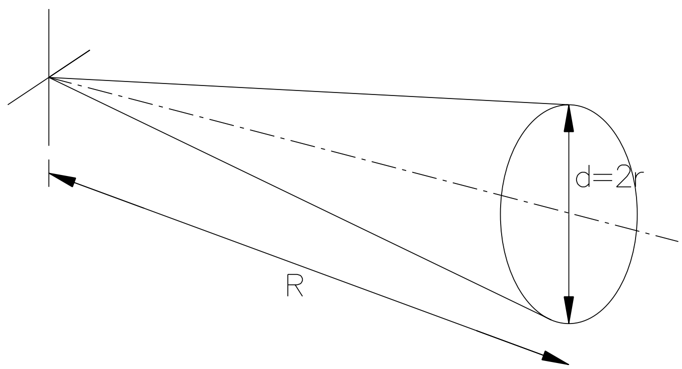

Figure 2: Geometry for Problem 1.3a. We want the spot diameter d for a given range R and beam divergence ϕ.

$$
\begin{aligned}
d = 2r = 2R \tan\left(\frac{\phi}{2}\right) &\approx \frac{2R\phi}{2} = R\phi \\
&\approx (4.5 \times 10^8)(1 \times 10^{-3}) = 4.5 \times 10^5 \text{ m} = 450 \text{ km}\,. \qquad (6a)
\end{aligned}
$$

b. ... if $\phi = 1$ μr??

$$d = 2r \approx R\phi = (4.5 \times 10^8)(1 \times 10^{-6}) = 4.5 \times 10^2 \text{ m} = 450 \text{ m}\,. \qquad (6b)$$

c. We want to design a beam collimator so that

$$\frac{\phi_{\text{out}}}{\phi_{\text{in}}} = \frac{1\ \mu\text{r}}{1 \text{ mr}} = 1 \times 10^{-3}\,. \qquad (7)$$

See Figure 3.

$$
\begin{aligned}
\frac{d_{\text{out}}}{d_{\text{in}}} &= 1 \times 10^3 \qquad (8a)\\
d_{\text{out}} &= 1 \times 10^3 \cdot (1 \times 10^{-3}) = 1 \text{ m}\,. \qquad (8b)
\end{aligned}
$$

We will need a lens (or mirror) with a diameter of *at least* 1 meter diameter. The focal length of the lens should be 2x (or more) the diameter of the lens (or else the lens will be prohibitively expensive). I will arbitrarily choose to make the lens diameter equal to 1.5 times the beam diameter.

$$d_2 = 1.5 d_{\text{out}} = 1.5(1.0) = 1.5 \text{ m}\,. \qquad (9)$$

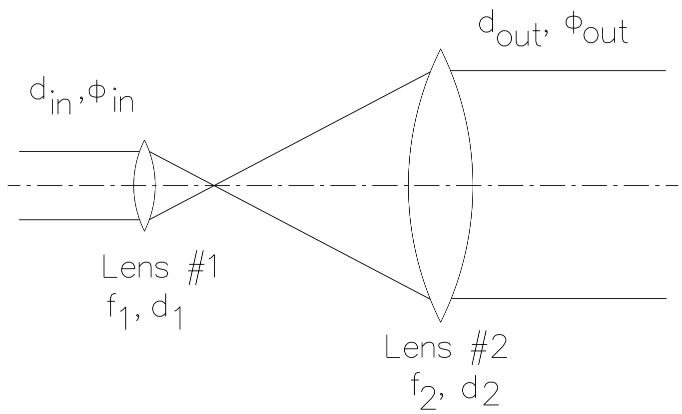

Figure 3: Beam expander for Prob. 1.3c.

The lens diameter d_1 should be at least 1.5 mm to accommodate the input beam diameter.

The focal lengths of the lenses should have the ratio

$$\frac{f_1}{f_2} = \frac{\phi_{\text{out}}}{\phi_{\text{in}}} = 1 \times 10^{-3} \tag{10}$$

If lens #1 has a focal length of 2 mm (arbitrarily chosen), then lens #2 will have a focal length of 2 m.

So we have

$$\text{Lens } \#1: \quad f_1 = 2 \text{ mm}; \quad d_1 = 1.5 \text{ mm} \tag{11a}$$

$$\text{Lens } \#2: \quad f_2 = 2 \text{ m}; \quad d_2 = 1.5 \text{ m}\,. \tag{11b}$$

Note: Lens #2 would be very expensive. It would be better implemented as a mirror. The design of the mirror system is beyond the scope of the course, but it *is* possible to make the required 1.5 m diameter mirror.

1.6. A plane wave at a wavelength of 10.6 μm illuminates a circular aperture of unknown diameter. The far–field pattern is measured at a distance of 10 m from the aperture. It shows that the first zero of the diffraction pattern is 1.3 m from the center of the pattern.

a. Calculate the diameter of the aperture.

b. Calculate the expected measured distance to the first zero of the diffraction pattern if the wavelength is changed to 514.5 nm through the use of an argon laser.

Solution: a. Given a spot radius of $r_{\text{wave}} = 1.3$ m at a range of $R = 10$ m, we want to find the size of the diffracting aperture. See Fig. 4.

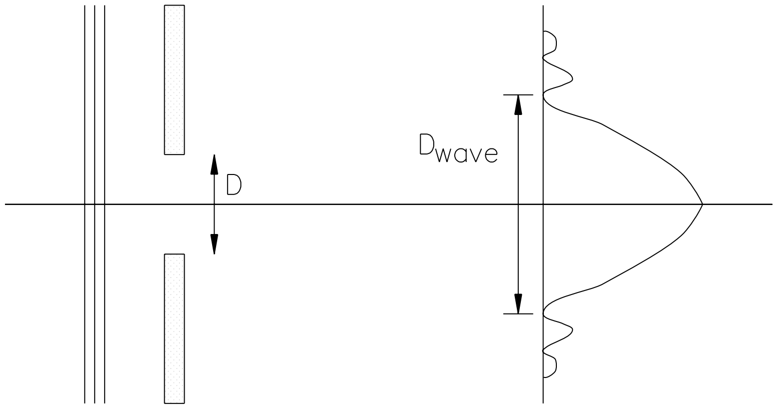

Figure 4: Problem 1.6.

Assuming that the observation plane is in the far-field, we have

$$D_{\text{wave}} = (2)(1.3) = 2.6 \text{ m} \tag{12a}$$

$$D = \frac{2.44\lambda R}{D_{\text{wave}}} = \frac{(2.44)(10.6 \times 10^{-9})(10)}{2.6} = 9.95 \times 10^{-5} \text{ m} = 99.5 \ \mu\text{m}\,. \tag{12b}$$

Checking that we are in the far-field, we want to show that

$$R \gg \frac{D^2}{\lambda} = \frac{(9.95 \times 10^{-5})^2}{10.6 \times 10^{-6}} = 9.34 \times 10^{-4} \text{ m} = 0.934 \text{ mm}\,. \tag{13}$$

We find that indeed $R = 10$ m is a lot greater than 0.934 mm, so our far-field assumption is correct.

b. If the wavelength changes for a given aperture, the spot size also changes. We have

$$\frac{D_{\text{wave}}(\lambda_1}{D_{\text{wave}}(\lambda_2} = \frac{\lambda_1}{\lambda_2} \tag{14a}$$

$$D_{\text{wave}}(\lambda_2) = \left(\frac{\lambda_2}{\lambda_1}\right) D_{\text{wave}}(\lambda_1) \tag{14b}$$

Hence, we find

$$\begin{aligned} D_{\text{wave}}(514.5 \text{ nm}) &= \left(\frac{514.5 \times 10^{-9}}{10.6 \times 10^{-6}}\right) D_{\text{wave}}(10.6 \ \mu\text{m}) \\ &= \left(\frac{514.5 \times 10^{-9}}{10.6 \times 10^{-6}}\right) (2.6) = 1.26 \times 10^{-1} \text{ m} \end{aligned} \tag{15}$$

and

$$R_{\text{wave}} = \frac{D_{\text{wave}}}{2} = 6.31 \times 10^{-2} \text{ m} = 6.31 \text{ cm}\,. \tag{16}$$

1.8. Light focusing I: A collimated light beam can be focused to a small spot located at the focal length of the focusing lens. The radius of the focused spot w_0 can be approximated by

$$w_0 \approx \frac{\lambda}{\pi}\frac{f}{w_1}\,, \tag{17}$$

where f is the focal length of the focusing lens and w_1 is the spot size of the beam entering the lens.

a. Calculate the spot size of a 5 cm diameter beam that is focused by a 10 cm focal length lens. The wavelength is assumed to be 1.06 μm.

b. Assuming that a power of 100 watts is uniformly distributed over a circle of the diameter of two spot sizes, calculate the irradiance of the beam both before the lens and at the focused spot.

c. Calculate the irradiance of the focused spot if the wavelength is changed to 10.6 μm. (Power is still 100 watts.)

d. ... if the wavelength is changed to 0.106 μm? (Power is still 100 watts.)

e. ... if a pulsed laser with a peak power of 10 MW is used at 10.6 μm?

We know that

$$w_0 \approx \left(\frac{\lambda}{\pi}\right)\left(\frac{f}{w_1}\right)\,. \tag{18}$$

a. We have $D = 5$ cm, so we expect $w_1 \approx D/2 = 2.5$ cm. We also have $f = 10$ cm and $\lambda = 1.06 \times 10^{-6}$ m.

$$w_0 \approx \left(\frac{\lambda}{\pi}\right)\left(\frac{f}{w_1}\right) = \frac{(1.06\times 10^{-6})(10\times 10^{-2})}{\pi(2.5\times 10^{-2}} = 1.35\times 10^{-6} = 1.35 \quad \mu\mathrm{m}\,. \tag{19}$$

b. In front of the lens we have

$$H_1 = \frac{P}{A} = \frac{4P}{\pi D_1^2} = \frac{(4)(100)}{\pi(5\times 10^{-2})^2} = 5.10\times 10^4\ \mathrm{W\cdot m^{-2}}\,. \tag{20}$$

and behind the lens

$$H_2 = \frac{P}{A} = \frac{P}{\frac{\pi D_2^2}{4}} = \frac{4P}{\pi(2w_0)^2} = \frac{(4)(100)}{\pi(2.7\times 10^{-6})^2} = 1.75\times 10^{13}\ \mathrm{W\cdot m^{-2}}\,. \tag{21}$$

c. If the wavelength is changed from $\lambda_1 = 1.06$ μm to $\lambda_2 = 10.6$ μm, we use

$$H \sim \frac{1}{w_0^2} \sim \frac{1}{\lambda^2}\,, \tag{22}$$

so

$$\frac{H(\lambda_1)}{H(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^2 = 100\,. \tag{23}$$

Hence,

$$H(\lambda_2) = \frac{1}{100} H(\lambda_1) = \frac{1.75 \times 10^{13}}{100} = 1.75 \times 10^{11} \text{ W} \cdot \text{m}^{-2} . \tag{24}$$

d. If the wavelength is changed to 0.106 μm, we use the same relations as in the previous portion of the problem:

$$\frac{H(\lambda_1)}{H(\lambda_2)} = \left(\frac{\lambda_2}{\lambda_1}\right)^2 = \frac{1}{100} \tag{25}$$

and, so,

$$H(\lambda_2) = 100 H(\lambda_1) = (100)(1.75 \times 10^{13}) = 1.75 \times 10^{15} \text{ W} \cdot \text{m}^{-2} . \tag{26}$$

e. If the peak power at $\lambda = 10.6$ μm is 1×10^7, we have

$$H_{\text{peak}} = \frac{P_{\text{peak}}}{\frac{\pi D^2}{4}} = \frac{(4)(1 \times 10^7)}{\pi (2w_0)^2} = \frac{(4)(1 \times 10^7)}{\pi (2 \cdot (1.35 \times 10^{-5}))^2} = 1.75 \times 10^{16} \text{ W} \cdot \text{m}^{-2} . \tag{27}$$

1.9. Light focusing II: If the light beam diameter ($\approx 2w_1$) is larger than the lens diameter D then we modify the equation of the spot size at the focus to

$$w_0 \approx 1.22 \frac{\lambda D}{f} .$$

The ratio f/D is the f/no. (called the "f-number") of the lens. A maximum f/no. is 0.5; a very expensive lens can approach values of 1.0 , and an ordinary lens will have an f/no. greater than 2.0. The fraction of the incident power that is intercepted by the lens and focused is $(D/2w_1)^2$.

a. Suppose that a 10 kW beam from a CO_2 laser ($\lambda = 10.6$ μm) is expanded to a 10 cm diameter and focused by a 2.5 cm diameter lens with a focal length of 25 cm. Calculate the irradiance of the incident beam and of the focused spot.

Solution: For a beam larger than the focusing lens, part of the beam does not pass through the lens and part of the power is lost (see Fig. 5). We have

$$w_0 = \frac{1.22 \lambda D_1}{f} = \frac{1.22\lambda}{\text{f/no}} . \tag{28}$$

We are given $\lambda = 10.6$ μm, $f = 25$ cm, $P = 10$ kW, $D_1 = 2.5$ cm, and $2w_1 = 10$ cm. The power transmitted through the lens is

$$\frac{P_{\text{lens}}}{P_{\text{incident}}} = \frac{D^2}{2w_{\text{incident}}^2} \tag{29}$$

and, so,

$$P_{\text{lens}} = \frac{(10 \times 10^3)(2.5 \times 10^{-2})^2}{(10 \times 10^{-2})^2} = 625 \text{ W} . \tag{30}$$

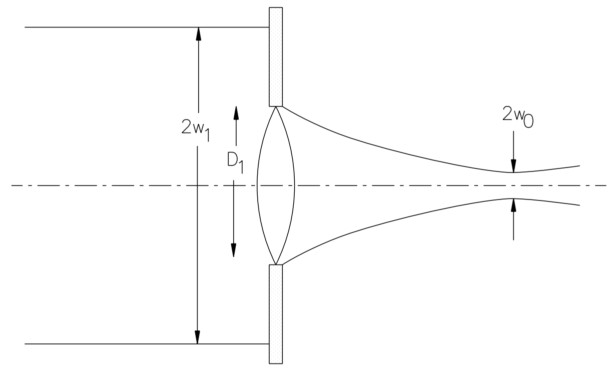

Figure 5: Geometry of lens focus for Problem 1.9. Beam width, $2w_1$, is bigger than lens diameter, D_1, in general. Focus spot diameter has minimum value of $2w_0$.

a. The irradiance H_2 is found as

$$H_2 = \frac{4P}{\pi D_{\text{focus}}^2} \tag{31}$$

where

$$D_{\text{focus}} \approx 2w_0\,, \tag{32}$$

so, we find

$$H_2 = \frac{4P}{4\pi w_0^2}\,. \tag{33}$$

The waist size w_0 is

$$w_0 = 1.22\frac{\lambda D}{f} = \frac{(1.22)(10.6\times10^{-6})(2.5\times10^{-2})}{25\times10^{-2}} = 1.293\times10^{-6}\ \text{m} = 1.293\ \mu\text{m}\,. \tag{34}$$

So,

$$H_2 = \frac{4P}{4\pi w_0^2} = \frac{(4)(625)}{4\pi(1.29\times10^{-6})^2} = 1.196\times10^{14}\ \text{W}\cdot\text{m}^{-2}\,. \tag{35}$$

1.10. Light focusing III: Another factor of importance in focusing applications is the *depth of focus*. This is the distance along the propagation axis that the beam "stays in focus", where the user has to define the tolerance on being "in focus". The depth of focus d_f is estimated by

$$d_f \approx \frac{2\lambda}{\pi}\left(\frac{f}{w_1}\right)^2\sqrt{\rho^2-1}\,, \tag{36}$$

where ρ is the fractional tolerance on the minimum focus size. For example, if the user wants to compute the distance over which the focused spot is within 10% of its minimum size, then $\rho = 1.1$. The equation can also be expressed in terms of the minimum spot size w_0,

$$d_f \approx \frac{2\pi}{\lambda}w_0^2\sqrt{\rho^2-1}\,. \tag{37}$$

a. Calculate the focused spot size w_0 and the depth of focus for a 5 cm diameter beam incident on a 10 cm diameter lens with a focal length of 25 cm. The beam is to be within 5% of its minimum size over its depth of focus and the wavelength is 488 nm.

b. Repeat the calculation if the focal length of the lens is doubled.

c. Using the focused spot size of Prob. 9a when the focal length was 25 cm, consider the case where a sheet of iron alloy is placed in the focal plane.

   i. Assuming that about 95% of the incident energy of 5 joules from a pulsed laser is reflected and that 5% is absorbed, calculate the absorbed energy.

   ii. If the volume receiving the energy is a hemisphere of alloy with a radius of two laser beam spot sizes (to allow for some conduction), calculate the volume that absorbs this energy.

   iii. Multiplying the absorption volume by the density of the material will give us the mass of the absorption volume. Calculate this mass if the density of alloy is 8 grams/cm^3.

   iv. The specific heat of a material is defined as the energy required to heat one gram (mass) by one degree C. The specific heat of the alloy is 0.525 joules/gram. Calculate the expected temperature rise (in degrees C) in the absorption volume.

<u>Solution:</u> We know that

$$d_f = \frac{2\lambda}{\pi}\left(\frac{f}{w_1}\right)^2\sqrt{\rho^2-1} = \frac{2\pi w_0^2}{\lambda}\sqrt{\rho^2-1}\,. \tag{38}$$

a. We are given that $2w_{\text{in}} = 5$ cm, $D = 10$ cm, $f = 25$ cm, $\rho = 1.05$ and $\lambda = 488$ nm (argon laser); hence

$$w_0 = \frac{\lambda f}{\pi w_{\text{in}}} = \frac{(488\times10^{-9})(25\times10^{-2})}{\pi(2.5\times10^{-2})} = 1.553\times10^{-6}\text{ m} = 1.553\ \mu\text{m}\,. \tag{39}$$

and

$$d_f = \frac{2\pi w_0^2}{\lambda}\sqrt{\rho^2-1} = \frac{2\pi(1.553\times10^{-6})^2}{488\times10^{-9}}\sqrt{(1.05)^2-1} = 9.94\times10^{-6}\text{ m}\,. \tag{40}$$

b. If f is doubled, then w_0 is doubled. Since $d_f \sim w_0^2$, d_f will be quadrupled.

$$d_f = 4(9.94)\quad \mu\text{m} = 39.76\ \mu\text{m}\,. \tag{41}$$

c. In Problem 9.a, we had a focused spot of $w_0 = 1.293\times10^{-6}$.

(1.10.c.i) We assert that

$$E_{\text{absorbed}} = \alpha E_{\text{incident}} = 0.05(5) = 0.25\text{ joules}\,. \tag{42}$$

(1.10.c.ii) We need to calculate the volume of a hemisphere with a radius of $2w_0$,

$$V = 0.5\left[\frac{4\pi r^3}{3}\right] = 0.5\left[\frac{4\pi(2w_0)^3}{3}\right] = 0.5\left[\frac{4\pi\left((2)(1.293\times10^{-6})\right)^3}{3}\right] = 3.62\times10^{-17}\text{ m}^3\,. \tag{43}$$

(1.10.c.iii) We now need to find the mass of the volume if the density of the material is 8 grams·cm^{-3} = 8×10^3 kg·m^{-3}.

$$M = \text{Density} \cdot V = (8 \times 10^3)(3.62 \times 10^{-17}) = 2.90 \times 10^{-13} \text{ kg}\,. \tag{44}$$

(1.10.c.iv) We now want to find the temperature rise, given that the specific heat of the material S is 0.525 joules·gram^{-1} = 525 j·(kg)$^{-1}$. We compute

$$\Delta T = \frac{E_{\text{absorbed}}}{SM} = \frac{0.25}{(525)(2.8 \times 10^{-13})} = 1.701 \times 10^9 \text{ C}\,. \tag{45}$$

Note: this is a considerable rise in temperature!